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Fracture parameters in Refel silicon carbide

The critical stress intensity factor in plane strain (K_{1c}) for ceramics is generally determined from three- or four-point bend tests. Summers *et al.* [1] have determined the correct equation to use for three-point bend tests but no similar evaluation has been made for four-point bending. The purpose of this note is to report and compare values of K_{1c} for Refel silicon carbide calculated by applying various analytical expressions to data obtained from four-point bend tests. In addition we have investigated the effects of precracking on K_{1c}.

The specimens were cut from blocks, lapped with 13 μm silicon carbide on a cast iron wheel, and cleaned in hydrochloric acid then alcohol. A notch 250 μm wide with a tip radius ~ 120 μm was machined in each specimen. In some cases, notched specimens were precracked by bending in a jig, heating to 600°C and quenching in water. The specimen dimensions are given in Fig. 1. The range of a/W ratios used in the present tests was 0.2 to 0.7. Four- rather than three-point bend tests were performed to avoid interaction between the contact stress and that at the root of the crack [2] and the difficulties of aligning the upper loading rod directly over the notch. In all tests a constant cross-head velocity of 0.05 cm min^{-1} was applied. Uncontrolled fracture always occurred, i.e. the load displacement curves had no tails.

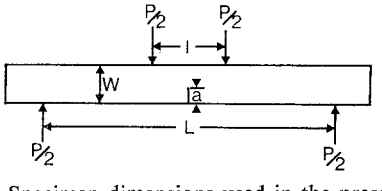

Figure 1 Specimen dimensions used in the present four-point bend tests. W = specimen height (5 mm), B = specimen width (2.5 mm), a = crack length (l to 3.5 mm), A = area of fracture face = $B(W - a)$, L = major span (40 mm), l = minor span (10 mm), d = bending arm = $(L - l/2)$ (15 mm), P = applied load.

Expressions for K_{1c} can be derived in various ways. According to the Griffith criterion, fracture occurs when the strain energy release rate $\partial U/\partial A$ reaches a critical value G_c, i.e.

$$\frac{\partial U}{\partial A} = G_c \cdot \tag{1}$$

G_c is related to the critical stress intensity factor K_c by the expression

$$G_c = \frac{\alpha K^2_c}{E} \cdot \tag{2}$$

In plane strain $K_c = K_{1c}$ and $\alpha = (1 - \nu^2)$ where ν is Poisson's ratio. G_c is obtained from conventional work of fracture tests for the case of controlled fracture, i.e. crack arrest, which is indicated by a tail on the load displacement curve. In this case U is estimated from the total area under the load displacement curve taking the projected surface area as A [3]. This method has the disadvantage that the values depend on a/W and the compliance of the test machine. An alternative treatment is as follows. Integration of Equation 1 gives:

$$U = G_c A + U_0 \,. \tag{3}$$

The energy constant U_0 relates to the energy losses not involved in the creation of new surfaces. Thus, G_c and hence K_{1c} can be determined graphically from a series of values of U and A for the case of uncontrolled fracture. The appropriate relationship is Equation A in Table I. A fracture mechanics approach has been used to derive a modified version of Equation A [4, 5]. This analysis has not previously been applied to four-point bending, but the appropriate expression (Equation B Table I) can be obtained similarly. The more usual fracture mechanics formulae (Equations C to E, Table I) have been derived by boundary collocation of a stress

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TABLE I

Equations used to determine K_{1c}	Reference	Definition of symbols
(A) $K_{1c} = \left[\frac{(U_f - U_0)E}{(1-\nu^2)A}\right]^{1/2} = \left\{\frac{[(P_f\Delta_f/2) - U_0]E}{(1-\nu^2)B(W-a)}\right\}^{1/2}$		P_f = applied load at fracture Δ_f = specimen deflection at fracture
(B) $K_{1c} = \left[\frac{(U_f - U_0)E}{(1-\nu^2)\eta A}\right]^{1/2} = \left\{\frac{[(P_f\Delta_f/2) - U_0]E}{(1-\nu^2)BW\phi}\right\}^{1/2}$	[4, 5]	$U_f = \frac{P_f\Delta_f}{2}$ = energy stored in the specimen at fracture
(C) $K_{1c} = Y_1\sigma a^{1/2} = \frac{3P_f d}{BW^2}a^{1/2}\left[1.99 - 2.47\left(\frac{a}{W}\right) + 12.97\left(\frac{a}{W}\right)^2 - 23.17\left(\frac{a}{W}\right)^3 + 24.8\left(\frac{a}{W}\right)^4\right]$	[6]	$\sigma = \frac{3P_f d}{BW^2}$ = The outer fibre tensile stress
(D) $K_{1c} = Y_2\sigma a^{1/2} = \frac{3P_f d}{BW^2}a^{1/2}\left[\frac{F(a/W)}{\frac{a}{W}\left(1-\frac{a}{W}\right)^3}\right]^{1/2}$	[7]	$F(a/W)$ is given in [7] η and ϕ are functions (related by the equation $\eta = \phi/[1 - (a/W)]$ which depend on the test method and a/W ratio. In the case of four-point bending, ϕ is given by the analytical expression:
(E) $K_{1c} = Y_3\sigma a^{1/2} = \frac{3P_f d}{BW^2}a^{1/2}\left[3.86 - 6.15\left(\frac{a}{W}\right) + 21.7\left(\frac{a}{W}\right)^2\right]^{1/2}$	[8]	$\phi = \frac{W}{Y_1^2 a}\left[\int Y_1^2\left(\frac{a}{W}\right)' \mathrm{d}\left(\frac{a}{W}\right)' + \frac{(3L-4d)}{18W}\right]$

function, Equation C being that recommended by the ASTM [6]. These three equations differ in the variation of the functions y_1 y_2 and y_3 with a/W, particularly when a/W exceeds 0.4. Equations C and D have previously been applied to ceramics, but Equation E has not, possibly because it is relatively unknown.

The values of K_{1c} calculated from Equations A to E are shown in Table II. As can be seen, for a given equation there is no difference in the K_{1c} values determined from notched or notched and thermally cracked samples, probably because small sharp cracks are introduced anyway at the notch tip during machining. Therefore, since precracking does not influence K_{1c} it is unnecessary in this material. This conclusion is different from that of McLaren *et al.* [10] who measured K_{1c} for both mechanically notched and precracked samples of Refel silicon carbide in three-point bending.

TABLE II Mean values of K_{1c} for Refel silicon carbide calculated using Equations A to E

	No. of tests	Equation				
		A	B	C	D	E
Machined notch	12	6.3	6.5	4.4	4.5	4.5
		± 1.0	± 0.9	± 0.1	± 0.1	± 0.1
Machined notch + thermal	7	5.3	5.7	4.4	4.5	4.2
shock crack		± 1.0	± 1.2	± 0.5	± 0.5	± 0.5
All specimens	19	5.9	6.1	4.4	4.5	4.4
		± 0.9	± 1.0	± 0.2	± 0.2	± 0.2

All errors quoted are standard errors calculated from Peter's formula [9].

No difference is seen between the mean values of K_{1c} calculated from Equations A and B for all specimens. This is because the mean values of A and ηA are within 10% of each other in this series of tests. However, these K_{1c} values are significantly higher than those obtained from Equations C to E for both types of notches because only the projected fracture area rather than the actual area can be measured. There is very little to choose between the mean values obtained from the other three methods i.e. Equations C to E. It must be emphasized that a basic tenet of fracture mechanics is that K_{1c} for any material is independent of specimen configuration and test method provided certain requirements are met [6]. Therefore, in the present case, to differentiate between Equations C to E it is necessary to examine the dependence of K_{1c} on specimen dimensions, e.g. a/W. Table III shows correlation coefficients [9] for a/W and the K_{1c} values obtained from Equations C to E.

TABLE III Correlation coefficients for K_{1c} and a/W

Equation no.	Correlation coefficient, r
C	0.43
D	0.44
E	0.11

Only for Equation E is K_{1c} essentially independent of sample dimensions. Using this equation the mean value of K_{1c} is 4.4 $\pm$ 0.2 MN m$^{-3/2}$ for the nineteen tests. This corresponds to a Griffith flaw size of 45 μm, using a mean value of $\sigma =$ 370 $\pm$ 11 MN m^{-2} obtained from ten measurements of bend strength on unnotched bars. As a further evaluation of Equation E, K_{1c} was calculated for six notched specimens with $B/W = 0.2$ compared with a value of 0.5 used previously (see Fig. 1). The mean was 4.8 $\pm$ 0.3 MN m$^{-3/2}$ which agrees well with the above value. Thus it is concluded that, for four-point bending, Equation E provides the best estimate of K_{1c}.

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